# On the Pólya problem of conversion for permanent to determinant

Alexander Guterman
Moscow State University

Two important functions in matrix theory, determinant and permanent, look very similar:

$$\det A = \sum_{\sigma \in S_n} (-1)^{\sigma} a_{1\sigma(1)} \cdots a_{n\sigma(n)} \quad \text{and} \quad \operatorname{per} A = \sum_{\sigma \in S_n} a_{1\sigma(1)} \cdots a_{n\sigma(n)}$$

here $A = (a_{ij}) \in M_n(\mathbb{F})$ is an $n \times n$ matrix and $S_n$ denotes the set of all permutations of the set $\{1, \ldots, n\}$.

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there exists a polynomial algorithm to compute the permanent. Due to this reason, starting from the work by Pólya, 1913, different approaches to convert the permanent into the determinant were under the intensive investigation.

Among our results we prove that if $n \geq 3$, and $\mathbb{F}$ is a finite field with $\operatorname{char} \mathbb{F} \neq 2$, then, no bijective map $T : M_n(\mathbb{F}) \to M_n(\mathbb{F})$ satisfies $\operatorname{per} A = \det T(A)$.