The Finite Element Method for the Porous Media Equation in 2D with Variable Exponent

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Abstract:
We study the following parabolic equation,

$$u_t = div(|u|^{\gamma(\mathbf{x})}\nabla u) + f(\mathbf{x}, t), \quad \mathbf{x} \in \Omega \subset \mathbb{R}^2,\ t \in]0, T].$$

In particulary we proved the convergence of the Finite Element Method (FEM) when applied to this equation. Since the problem may be of degenerate type, we utilize an approximate problem, regularized by introducing a parameter ε. We prove, under certain conditions on γ and f, that the weak solution of the approximate problem converges to the weak solution of the initial problem, when the parameter ε tends to zero. Discrete solutions are built using the FEM and the convergence of these for the weak solution of the initial problem is proved.
We also study the application of the Moving Mesh Method to this problem, considering it a free boundary problem with a time dependent domain.
The spatial discretization is defined by a triangulation of the domain. In each finite element, the solution is approximated by piecewise polynomial functions of degree $gp \geq 1$ using Lagrange interpolating polynomials in area coordinates. The vertices of the triangles move according to a system of differential equations which is added to the equations of the problem. The resulting system is converted into a system of ordinary differential equations in time variable, which is solved using a suitable integrator. The integrals that arise in the system of ordinary differential equations are calculated using the Gaussian quadrature. Finally, we present some numerical results of application.

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