Moving Finite Element Method: Application to Multidimensional Moving Boundary Models

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Abstract

Problems with moving boundaries, often called Stefan problems, appear in many areas. Analytical solutions are available for certain two-phase problems in two space dimensions, this was the motivation for developing numerical algorithms.

The numerical simulation of mathematical models of dynamic two-phase systems described by partial differential equations (PDE) is a difficult problem, particularly when a moving interface is involved and/or the solution develops steep moving fronts. We have developed a numerical algorithm for time dependent PDE based on the moving finite element method (MFEM). The adaptive grid methods are widely used to overcome the difficulties in numerically solve these type of problems. The MFEM is an adaptive grid method, especially designed to simulate time dependent problems with solutions that exhibit sharp transition layers. In the MFEM, originally developed by Miller and Miller [1], the solution is approximated by a piecewise linear function depending on the nodal amplitudes and on the nodal positions. So, the MEFM automatically relocates nodes in order to concentrate them in regions where the solution has steep gradients. Coimbra [2] implemented an algorithm based on the MFEM with higher order basis functions for the solution of time dependent PDE systems in two space dimensions. The MFEM generates the solution and an adaptive spatial mesh for each dependent variable by minimizing the PDE residual with respect to these unknowns.

In this work, the formulation of the MFEM proposed by Coimbra was generalized to the numerical simulation of two phase systems with a moving interface. We pay particular attention to the implementation of the interface boundary conditions. Special domain decomposition is implemented by the introduction of moving nodes defining a piecewise linear approximation of the moving boundary.

Results will be presented to demonstrate the accuracy and the effectiveness of the method, including comparisons with the analytical solutions.

References

[1] Miller, K. and R. N. Miller, Moving Finite Elements. *SIAM J. Numer. Anal.*, Vol **18/6**, 1019-1032, 1981.

[2] Coimbra, M. C, Sereno, C. A. And Rodrigues A. E., Moving Finite Element Method: Applications to Science and Engineering Problems. *Computers and Chemical Engineering*, Vol **28/5**, 597-603, 2004.

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