UNIVERSIDADE BEIRA INTERIOR

Departamento de Matemática
Centro de Matemática e Aplicações

Seminar

Automorphisms of infinite-dimensional flag varieties

Mikhail Ignatev

Samara National Research University, Jacobs University Bremen

Abstract: I would like to pose and solve the problem of computing $\mathrm{Aut}\, X$ for a class of homogeneous ind-varieties X. This is the class of ind-varieties of generalized flags introduced by that name I. Dimitrov and I. Penkov. These ind-varieties can be defined as G/P where G is one of the ind-groups $\mathrm{SL}(\infty) = \varinjlim \mathrm{SL}(n)$, $\mathrm{SO}(\infty) = \varinjlim \mathrm{SO}(n)$, $\mathrm{Sp}(\infty) = \varinjlim \mathrm{Sp}(2n)$ and P is a splitting parabolic subgroup, i.e., a subgroup for which the intersections $P \cap \mathrm{SL}(n)$, $P \cap \mathrm{SO}(2n)$, $P \cap \mathrm{SO}(2n+1)$, $P \cap \mathrm{Sp}(2n)$ are parabolic subgroups of $\mathrm{SL}(n)$, $\mathrm{SO}(2n)$, $\mathrm{SO}(2n+1)$, $\mathrm{Sp}(2n)$ for all n, respectively. There also exists a flag realization of the ind-varieties G/P as above. The main idea of that approach is that one designates certain chains of subspaces in the natural representation V of $\mathrm{SL}(\infty)$ as generalized flags, and then defines an ind-variety of generalized flags as the ind-variety of generalized flags which differ only "slightly" from a fixed generalized flag W in V. One then shows that the so obtained ind-variety is isomorphic to G/P for $G = \mathrm{SL}(\infty)$ and some splitting parabolic subgroup $P \subset G$.

The main result is the explicit determination of the group $\mathrm{Aut}\, X$ for an arbitrary ind-variety of, possibly isotropic, generalized flags. A notable feature is that the answer is very different form the ind-groups $P\mathrm{GL}(\infty)$, $P\mathrm{SO}(\infty)$, or $P\mathrm{Sp}(\infty)$, and it is presented in terms of Mackey groups. Such a group is defined in terms of a nondegenerate pairing of vector spaces $T \times R \to \mathbb{C}$, and is a subgroup of the group of all linear operators $\varphi\colon T \to T$ for which the dual operator φ^* determines a well-defined automorphism $\overline{\varphi}\colon R \to R$. This definition of Mackey group is inspired by G. Mackey's dissertation. If T and R are finite dimensional, then the Mackey group is nothing but $\mathrm{GL}(T) \cong \mathrm{GL}(R)$. The group known as Japanese $GL(\infty)$ is a Mackey group and plays a crucial role in this work.

An ind-grassmannian is an ind-variety of generalized flags for which the fixed generalized flag consists of a single proper subspace $W \subset V$. For $\dim W = \mathrm{codim}_V W = \infty$, the ind-grassmannian is isomorphic to the Sato grassmannian. For the Sato grassmannian (which is the most interesting ind-grassmannian) the main result implies that its automorphism group is isomorphic to the projectivization of the connected component of the identity in the group Japanese $\mathrm{GL}(\infty)$.

(**continues on the next page**)

Dia: 29 de Abril de 2022, às 15h

Local: Sala de Reuniões, Departamento de Matemática, UBI

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Funded by the Portuguese Government through the FCT - Fundação para a Ciência e a Tecnologia under the project PEst-OE/MAT/UI0212/2013

I would like to point out that $\mathrm{Aut}\,X$ depends essentially on the ind-variety X, despite the fact that all X are homogeneous spaces for the same group $\mathrm{SL}(\infty)$ (or, respectively, $\mathrm{SO}(\infty)$, $\mathrm{Sp}(\infty)$). This is in contrast with the finite-dimensional case in which the connected component of the identity in the automorphism group of a variety $\mathrm{SL}(n)/P$ (respectively, $\mathrm{SO}(2n)/P$, $\mathrm{SO}(2n+1)/P$ or $\mathrm{Sp}(2n)/P$) depends only on n and not on the choice of P.

The talk is based on our joint work with Ivan Penkov [M. Ignatev, I. Penkov. Automorphism groups of ind-varieties of generalized flags. Transformation Groups, to appear; arXiv: `math.AG/2106.0099`.]

Dia: 29 de Abril de 2022, às 15h

Local: Sala de Reuniões, Departamento de Matemática, UBI

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Funded by the Portuguese Government through the FCT - Fundação para a Ciência e a Tecnologia under the project PEst-OE/MAT/UI0212/2013