An Application of Brown's Lemma to Tiling Theory.

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The main idea of Ramsey theory is that arbitrarily large sets cannot avoid a certain degree of "regularity". This is exemplarily illustrated by Gallai's theorem, a multidimensional version of the seminal van der Waerden's theorem, which asserts that, given a finite coloring of $\mathbb{Z}^n$, any finite subset F of $\mathbb{Z}^n$ has a monochromatic homothetic copy $\lambda F + \vec{t}$, for some real number λ. A not so famous Ramsey-type result is the so called Brown's lemma: in the setting of Gallai's theorem, one can take any λ once "bounded perturbations" in the structure of the homothetic copies of F are allowed, in other words, any finite coloring of $\mathbb{Z}^n$ produces monochromatic *piecewise syndetic* sets. In this talk we describe an application of a topological dynamical version of this result to (euclidean space) tiling theory and compare it with previous applications of topological dynamical systems results to tiling theory by De La Llave & Windsor and Radin & Wolff.